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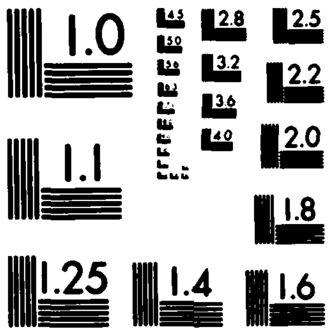

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Annual Technical Report

Project Title: Extreme Values of Queues, Point Processes and Stochastic Networks

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Extreme Values of Queues, Point Processes and Stochastic Networks

During the last year we worked on four research topics. Our progress on these topics is described in the following discussion.

1. Modeling of Stochastic Flows in Networks: Compound Poission Approximations

Our most significant accomplisment last year was the development of compound Poisson approximations for random variables and Point processes. Such approximations are instrumental in the modeling of stochastic flows in networks. Being fundamental in nature, our results apply to other settings as well. The following papers described our work; further discussion is given below.

Serfozo, R. F. (1985). Compound Poisson Approximations for Sums of Random Variables. To appear in Ann. Probability.

Serfozo, R. F. (1985). Partitions of Point Processes: Poisson Approximations. To appear in Stochastic Processes and Their Applications.

Compound Poisson Approximations for Sums of Point Processes. A basic theme in probability is the characterization of the behavior of sums of random variables and point processes. Many physical quantities can be viewed as a sum of a large number of small quantities (e.g. an SAT score is the sum of scores from individual questions, or a company's revenue in a month is the sum of the revenues from its many sales). Moreover, any random sequence S_n can be viewed as the sum of its increments:

$S_n = \sum_{k=1}^{n} (S_k - S_{k-1}) - S_0$. The classical central limit theorem for a sum S_n of independent identically distributed random variables asserts that the distribution of S_n, for large n, is approximately normal, and the quality of this approximation is described by the Berry-Esseen inequality. It is also known that S_n under slightly different conditions, may be approximately Poisson, compound Poisson or infinitely divisible, and there are known error estimates for the Poisson approximation.

The Poisson approximation is frequently used in the operational analysis of telecommunications networks. For instance, the number of telephone calls that arrive to a switching station in an hour from a large number of subscriber lines, as shown below, is typically modeled as a Poisson random variable. More generally, the flow of calls over time from each subscriber is viewed as a "thin" point process and the sum of these point processes that enters the station is modeled as a Poisson process.

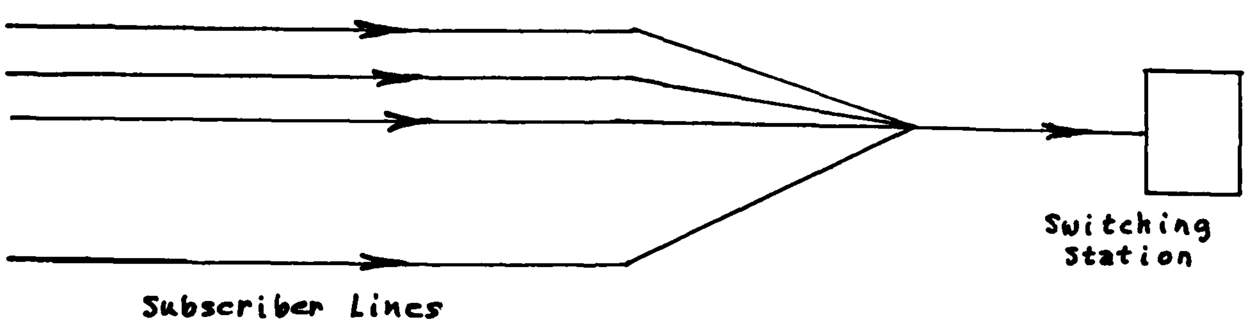

Figure 1. Flows in a Telecommunications Network

This type of merging or summation of point processes occurs in other networks such as (i) flows of data packets in computer networks, (ii) flows of material and parts in automated production plants, and (iii) flows of goods in distribution networks. (These are the principle application areas for our results.) Although Poisson processes are used for modeling flows in these networks, they are inappropriate when the flows have certain natural groupings of points (e.g. a series of data-packets consitutte a message, or a group of parts constitute a delivery). In such instances, which are evidently more common than not, a compound Poisson model may be more appropriate than a Poisson model. This raises the questions: under what conditions can a sum of random variables or point processes be approximated by compound Poisson random variables or point processes? This question is what motivated our research.

During the last twenty years, several ad hoc theorems had been proved on the convergence of sums of independent random variables to compound Poisson variables, but little was known about the error in their attendant approximations. D. Freedman (1973) gave some examples that seemed to imply that one could not develop compound Poissosn approximations that would be as natural or universal as normal or Poisson approximations.

In spite of this dire evidence, we have been fortunate enough to develop such approximations. We have found rather general conditions under which sums of dependent random variables or sums of dependent point processes are asymptotically compound Poisson. More important, we have established bounds on the errors involved in these approximations. Our

results are applicable, for instance, for constructing compound Poisson models of merging of flows in networks as described above. These models could be used in conjunction with queueing models to analyze the delay or throughput of the flows. Another major application of our results is described next.

Partitions of Point Processes. The preceding discussion was on the merging of stochastic flows in networks. Another related operation is the partitioning of a single flow into many subflows as shown below.

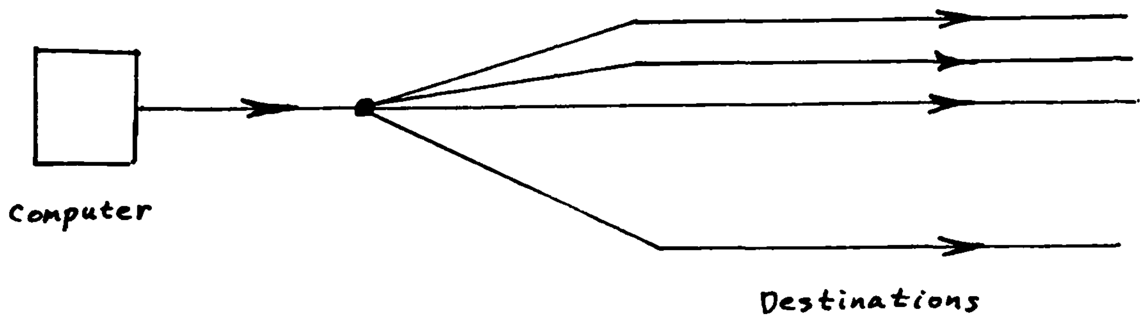

Figure 2. Subflows in a Computer Network

Here a stochastic flow of computer data packets on a network line is entering a computer that directs the packets to several other computers depending on the packets' respective instructions. In other words, the initial flow is randomly partitioned into several subflows. When the number of subflows is large so that each subflow is relatively thin, then one would suspect that the subflows may be modeled as multi-variate Poisson or compound Poisson point processes. Using the results described above, we have been able to shed light on this phenomena. We have found several types of random partitions whose resulting subflows are

approximately Poisson or compound Poisson, and we have obtained bounds on the errors in these approximations.

Partitions of point processes, like sums, are fundamental to a variety of contexts other than networks. For instance, consider a point process over time in which each point has one of several attributes (e.g. insurance claims over time may be categorized as small, medium or large in size), then the numbers of points with these attributes form a partition of the parent process. Our results are useful for analyzing the dependency among such subflows as well as the characteristics of each subflow.

2. Extremal Problems in Stochastic Networks

We have obtained a family of bounds for the distributions of certain generic random variables associated with networks. These random variables represent critical path lengths in PERT networks, maximum flows in networks, and lifetimes of systems. This work is documented in:

Weiss, G. (1985). Stochastic Bounds on Distributions of Optimal Value Functions with Applications to PERT, Network Flows and Reliability. Technical report, Georgia Tech.

Description of the Study. We consider a network with nodes $\{1,\ldots,n\}$ and random variables $X_1,\ldots,X_n$ associated with the nodes. Let $I_1,\ldots,I_k$ denote the sets of paths and let $J_1,\ldots,J_\ell$ denote the sets of cuts of the network. We focus on the following random variables.

(a) Critical Path of a PERT network: The nodes represent

activities, the X_i are activity durations, and the network structure depicts the precedence constraints. The critical path length is the shortest time needed to complete the project, namely

$$M = \max_{1 \leq j \leq k} \sum_{i \varepsilon I_j} X_i .$$

(b) Maximal Flow in a Network: The nodes represent pipelines and the X_i are maximal flow capacities. The maximal flow through the network from source to sink is

$$L = \min_{1 \leq j \leq \ell} \sum_{i \varepsilon J} X_i .$$

(c) Reliability of a System: The nodes represent components and the X_i are their lifetimes. The system lifetime is

$$T = \max_{1 \leq j \leq k} \min_{i \varepsilon I_j} X_i = \min_{1 \leq j \leq \ell} \max_{i \varepsilon J_j} X_i .$$

It is generally impossible to obtain tractable expressions for the distributions of M, L, T in terms of the joint distribution P of the $X_1, \ldots, X_n$. Consequently, it is natural to seek partial information or bounds on M, L, T. In this regard, we consider worse-case bounds of M, L, T. Specifically, we address the question: What joint distributions P on the $X_1, \ldots, X_1$ solve the following optimizaton problems

$$\max_P E(M - x)^+, \qquad \max_P E(L - x)^-$$

$$\max_{P} P(T > x), \qquad \sup_{P} P(T < x),$$

where the optimization is over all joint distributions P with the fixed marginal distributions $F_1,\ldots,F_n$? We answer this question by presenting mathematical programming algorithms for optimizing P in these problems. This gives us worse-case networks in which the distributions of M, L, T can be computed. These distributions are then bounds for M, L, T in any network.

3. Optimization of Queueing Systems

Two major problem areas in the optimization of queueing systems are as follows:

Optimal Design of Queueing Systems. In designing a service system involving queueing, one typically is able to choose some of the basic parameters of the system (such as numbers of servers or arrival and service rates) from a range of possible values. The problem is to select the parameters so as to minimize the total cost of the system, including the operational cost of the system over its lifetime. This is a static optimization in that the parameters are chosen at the inception of the system and are thereafter fixed for the system's lifetime.

Optimal Dynamic Control of Queueing Systems. In some queueing systems, the basic parameters can be changed dynamically as the queues evolve. For instance in telecommunications systems, the service rate or numbers of servers change as the queue lengths change. The problem is to determine a policy for dynamically regulating the system parameters, based on the queue length, so as to minimize the total cost of operating the system.

We have begun work on several problems in these areas; our progress on these is discussed below. This work compliments our analysis of extreme values of queues discussed in Section 4 in that here we are seeking economical ways to control or dampen extremes of queus.

Optimal Idle and Inspection Periods for M/G/1 Queues

In a standard M/G/1 queueing system, a Poisson stream of customers arrives to a single server who serves them on a first-come-first-serve basis and the service times are independent and identically distributed. In this system, the server is alwasy available for service. In actual systems, however, a server may have to be absent periodically for other duties or for rejuvenation (e.g. a computer may do file maintenance in addition to its standard processing of jobs). In such systems, the customers are served intermittently rather than continuously. Intermittant service is also characteristic of service systems in which short queues are tolerable or when short busy periods for servers is uneconomical. In designing such a system, a natural question is: How long should the server be absent without serving customers and how large should the queue be before the server starts serving customers?

We have solved this problem for an M/G/1 queue that operates under a (T,N)-policy described as follows. Whenever the system becomes empty, the server is idle for a time T and then it inspects the queue continuously without serving customers until there are N customers waiting - thereupon the server is activated and serves customers continuously until the system becomes empty. This idle-inspection-service cycle is repeated indefinitely. There are costs for inspecting the queue, for activating and running the server, and for holding customers in the system. We have

developed a nonlinear programming model for determining the parameters (T,N) that minimize the average cost. This is documented in the following paper.

Kim, S. S. and R. F. Serfozo (1985). Optimal Idle and Inspection Periods for M/G/1 Queues. Technical Report, Georgia Tech.

Optimal Control of Networks of Queues. Service systems in manufacturing and telecommunications usually involve random flows of customers among a network of queueing systems.

We have begun a study of the dynamic control of a network in which the service rates at the nodes and the routing of the customers through the network are subject to control each time a customer moves in the network. Whenever a customer moves, the queues in the entire network are observed and, based on the observation, the service rates and routing probabilities are selected until the next customer movement. This is repeated indefinitely. Our aim is to establish the existence of certain natural monotone optimal policies in which the service rates are increasing functions of the queue lengths and the routing probabilities have monotonicity properties such that large queues are avoided. The knowledge of the existence of such policies leads to efficient computational procedures for optimal policies. Furthermore, monotone policies are more natural for implementation in actual systems.

Our approach to this problem area is as follows. We characterize the queueing network process as a multi-dimensional Markov process whose transitions are determined by a family of "transition operators". (As a simple example, a birth and death process has two operators: an upward unit jump and a downward unit jump.) We first establish certain optimal

monotonicity properties for these operators, and then translate these into monotonicity properties for the parameters under control. The analysis involves transforming the Markov process into a simpler process and introducing and exploiting the notion of convexity and submodularity of functions with respect to the transition operators. We plan to start documenting our results on this next year.

4. Extreme Values of Queues and Point Processes

Although much of our effort this year has been spent on this topic, we have not reached the documentation stage yet. The research is proceeding along the lines of our proposal, which need not be reviewed here.

There are several technical issues that we are striving to understand more fully: (i) Our major results show that queueing processes have an asymptotic extreme-value distribution that is different from the three classical ones. To shed light on why this is so, we are attempting to prove our results by another approach, possibly via limits of diffusion processes. (ii) We are seeking a more complete characterization of the types of service times in queues for which our results apply. (iii) We are attempting to obtain necessary as well as sufficient conditions for our limit properties of queues and point processes.

We will give a more extensive review of this work in our next progress report.

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Work progressed on four topics. In modeling of stochastic flows in networks, compount Poisson approximations for random variables and point processes were developed. In extremal problems in stochastic networks, a family of bounds for the distributions of certain generic random variables were obtained. In optimization of queueing systems, progress was made in determining ways to control or dampen extremes of queues. Finally, research was begun on extreme values of queues and point processes.

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